AD-A078 640 TEXAS UNIV AT AUSTIN F/G 12/1
SMOOTHING AND APPROXIMATION OF MULTIVARIATE FUNCTIONS.(U)
OCT 79 E W CHENEY DAAG29-77-G-0215
UNCLASSIFIED ARO-14900.9-M NL

1 OF 1
AD
A078640

END
DATE
FILMED
1-80
DDC

MICROCOPY RESOLUTION TEST CHART
NATIONAL BUREAU OF STANDARDS-1963-A

ARO 14900.9-M

12

ADA078640

SMOOTHING AND APPROXIMATION OF MULTIVARIATE FUNCTIONS

LEVEL II

FINAL REPORT

E. W. CHENEY

OCTOBER 31, 1979

U. S. ARMY RESEARCH OFFICE

GRANT NUMBER DAAG29-77-G-0215

THE UNIVERSITY OF TEXAS AT AUSTIN

Accession For	
NTIS GRA&I	X
DDC TAB	
Unannounced	
Justification	
By	
Distribution/	
Availability Codes	
Dist.	Avail and/or special
A	

79 12 17 069

UNCLASSIFIED

SECURITY CLASSIFICATION OF THIS PAGE (When Data Entered)

REPORT DOCUMENTATION PAGE

READ INSTRUCTIONS BEFORE COMPLETING FORM

1. REPORT NUMBER

2. GOVT ACCESSION NO.

3. RECIPIENT'S CATALOG NUMBER

4. TITLE (and Subtitle)

6 SMOOTHING AND APPROXIMATION OF MULTIVARIATE FUNCTIONS.

5. TYPE OF REPORT & PERIOD COVERED

FINAL
9/10/77 - 9/9/79

6. PERFORMING ORG. REPORT NUMBER

7. AUTHOR(s)

10 E. W. CHENEY

8. CONTRACT OR GRANT NUMBER(s)

15 DAAG29-77-G-0215 kw

9. PERFORMING ORGANIZATION NAME AND ADDRESS

The University of Texas at Austin
Austin, Texas 78712

10. PROGRAM ELEMENT, PROJECT, TASK AREA & WORK UNIT NUMBERS

12 10

11. CONTROLLING OFFICE NAME AND ADDRESS

U. S. Army Research Office
Post Office Box 12211
Research Triangle Park, NC 27709

11 31

12. REPORT DATE

Oct[redacted]979

13. NUMBER OF PAGES

8

14. MONITORING AGENCY NAME & ADDRESS (if different from Controlling Office)

15. SECURITY CLASS. (of this report)

Unclassified

15a. DECLASSIFICATION/DOWNGRADING SCHEDULE

16. DISTRIBUTION STATEMENT (of this Report)

Approved for public release; distribution unlimited.

9 Final rept. 14 Sep 77 - 9 Sep 79,

17. DISTRIBUTION STATEMENT (of the abstract entered in Block 20, if different from Report)

NA 18 ARO 19 14900.9-M

18. SUPPLEMENTARY NOTES

The view, opinions, and/or findings contained in this report are those of the author(s) and should not be construed as an official department of the Army position, policy, or decision unless so designated by other documentation.

19. KEY WORDS (Continue on reverse side if necessary and identify by block number)

20. ABSTRACT (Continue on reverse side if necessary and identify by block number)

This is a summary of work performed under the subject grant. An outline of the contents of each technical report is included, as is a comprehensive bibliography.

DD FORM 1 JAN 73 1473 EDITION OF 1 NOV 65 IS OBSOLETE

UNCLASSIFIED

SECURITY CLASSIFICATION OF THIS PAGE (When Data Entered)

347 800

mt

SMOOTHING AND APPROXIMATION OF MULTIVARIATE FUNCTIONS

Grant number DAAG29-77-G-0215, titled "Smoothing and Approximation of Multivariate Functions," was made to The University of Texas at Austin on 10 September 1977 and expired 9 September 1979. Principal investigator was E. W. Cheney, Professor of Mathematics.

The research results from this grant are reported in 8 technical papers which have been or will be published in the open scientific literature. They have also been issued in Preprint form at the Center for Numerical Analysis, University of Texas. At the end of this report, technical summaries of these papers will be presented.

The grant supported the principal investigator during four months in the summers of 1978 and 1979. In addition, the grant supported one Ph.D. student, John Respess, at a rate of half-time, during two academic years. Mr. Respess' dissertation will be completed in 1980. The grant provided a small amount of supporting service through the Center for Numerical Analysis. Several mathematicians from abroad who worked with the principal investigator were enabled to visit the University of Texas with funds from this grant. Finally, some travel funds from the grant enabled the principal investigator to accept an invitation to lecture at the Third Biannual Dundee Conference on Numerical Analysis (Dundee, Scotland, June 1979).

The goal of the research project was to proceed as far as possible in the study of a certain class of practical approximation problems. Namely, the problem is to reduce the complexity of multivariate functions by representing them precisely or approximately by combinations of univariate functions. A prototype problem is that of finding a best approximation to a function of two variables, $f(x,y)$, by a sum $g(x) + h(y)$. The prototype problem is thoroughly understood in the case that the functions involved are continuous and an approximation in the uniform sense is needed.

The general problem of approximating $f(x,y)$ by

$$(1) \qquad \sum_{i=0}^{n} a_i(x)y^i + \sum_{i=0}^{m} b_i(y)x^i$$

with undetermined continuous coefficient functions $a_i(x)$ and $b_i(y)$ is very poorly understood, and our efforts have been directed towards it.

A typical practical computing problem which leads to this type of approximation is the integral equation

$$u(x) = \int_0^1 f(x,y)\,u(y)\,dy \qquad (2)$$

in which $u(x)$ is an unknown function and the kernel function $f(x,y)$ is given. The theory shows that $u(x)$ can be determined easily if $f(x,y)$ is first approximated accurately in the form (1). The theory encompasses, of course, more general equations than (2) and more general forms than (1).

Research Reports by E. W. Cheney Supported by the Grant

1. "On Simultaneous Chebyshev Approximation" (with G. M. Phillips, J. H. McCabe), C A Report 131, February 1978. 8 pages. To appear in Journal of Approximation Theory 26 (1979).

This paper deals with the problem of minimizing an expression

$$\max_x \int |f(x,y) - g(x)|\,dy$$

when f is given and g ranges over a set of continuous functions. Several characterizations of the optimal function g are given. These generalize theorems previously proved in the literature. This paper also contains a bibliography of 30 references to works on "simultaneous approximation."

2. "On the Approximation of a Bivariate Function by Sums of Univariate Functions" (with W. A. Light), CNA Report 140, August 1978. 27 pages. To appear, Journal of Approximation Theory.

This paper offers a completely new proof of convergence of the Diliberto-Straus Algorithm for uniform approximation of a continuous bivariate function by sums of continuous univariate functions. Also, a special analysis of the discrete case (which is of importance in scaling problems of matrices) is given. In particular, the functions which have unique best approximations are completely characterized.

3. "On the Algorithm of Diliberto and Straus for Approximating Bivariate Functions by Univariate Ones" (with M. V. Golitschek). Numerical Functional Analysis and Optimization 1 (1979), 341-363. Also, CNA Report 141, August 1978.

This paper gives results on the efficiency of the Diliberto-Straus Algorithm for obtaining best approximations of the form

$$f(x,y) \approx g(x) + h(y)$$

using the Tchebycheff norm. For example, three steps produce an approximation within 50% of the ultimate final accuracy. Examples can be constructed, however, showing arbitrarily slow linear convergence. In the discrete case, best approximations possessing certain extra, favorable properties are shown to exist.

4. "The Approximation of Bivariate Functions by Sums of Univariate Ones Using the L_1-Metric" (with W. A. Light, T. McCabe, G. M. Phillips). CNA Report 147, December 1978, 16 pages. Submitted for publication.

This paper establishes the existence of best approximations

$$f(x,y) \approx g(x) + h(y)$$

in an L_1-setting. Thus the norm employed is the L_1-norm, and the functions g and h can be arbitrary integrable functions. It is also proved, by an example, that the algorithm of Diliberto-Straus does not produce best approximations in the L_1-norm.

5. "Multivariate Approximation with Tensor-product Spaces" (with W. A. Light). CNA Report 153, November 1979. 25 pages. Being readied for publication.

This report establishes the existence of best approximations to a function $f(x,y)$ by functions of the form

$$\phi(x,y) = \sum_{i=1}^{n} a_i(t)\, g_i(s) + \sum_{i=1}^{m} b_i(s)\, h_i(t)$$

when (1) the functions $g_i(s)$ and $h_i(t)$ are given as essentially bounded measurable functions; (2) the functions $a_i(t)$ and $b_i(s)$ are allowed to range over all integrable functions; and (3) the criterion for the approximation is the L_1-norm.

Thus, we want to make this measure of the error small:

$$\iint |f(x,y) - \phi(x,y)|\,dx\,dy$$

6. "Approximation Theory in Tensor-product Spaces" (with John Respess).

This is on-going work connected with Mr. Respess' Ph.D. research. A representative result states that each continuous function of two variables, s and t, possesses a best approximation of the form $y(s)h(t) + \sum_{i=1}^{n} x_i(t)g_i(s)$ if h, $g_1,\ldots,g_n$ are continuous, h has no zeros, $\{g_1,\ldots,g_n\}$ is a Tchebycheff set, and the functions y, $x_1,\ldots,x_n$ are free to range over all possible continuous functions.

7. "Approximating Multivariate Functions by Combinations of Univariate Ones," Proceedings of the 1979 Army Numerical Analysis and Computers Conference, El Paso, Texas, February, 1979, pp. 433-440.

This is an expository report, explaining several practical situations in which a multivariate function must be approximated by combinations of univariate ones. Progress on the existence of best approximations and on the algorithmic aspects of this branch of numerical analysis is outlined.

8. "Best Approximation in Tensor Product Spaces," CNA Report 151, August 1979, 8 pages. To appear, Proceedings of Dundee Conference on Numerical Analysis.

This is an expository account of the problem of representing multivariate functions by combinations of univariate ones. Applications are cited in the fields of numerical treatment of partial differential equations, numerical solution of integral equations, and preconditioning of matrices.

BIBLIOGRAPHY

Atkinson, K., "A Survey of Numerical Methods for the Solution of Fredholm Integral Equations of the Second Kind," SIAM, Philadelphia, 1976.

Attlestan, B., and F. Sullivan, "Iteration with Best Approximation Operators," Rev. Roum. Math. Pures Appl., XXI, No. 2 (1976), 125-131.

Aumann, G., "Über approximative nomographie, I and II," Bayer. Akad. Wiss. Math.-Nat. Kl. S.B. 1958, 103-109; ibid., 1959, 103-109.

Bank, R., and D. J. Rose, "Marching Algorithms for Elliptic Boundary Value Problems. I: The Constant Coefficient Case," SIAM J. on Numer. Anal. 14 (1977), 792-829.

Bank, R., "Marching Algorithms for Elliptic Boundary Value Problems. II: The Variable Coefficient Case," SIAM J. on Numer. Anal. 14 (1977), 950-970.

Bank, R., "An Automatic Scaling Procedure for a d'Yakanov-Gunn Iteration Scheme," CNA Report 142, Center for Numerical Analysis, UT Austin, August 1978. To appear, Linear Algebra and Its Applications.

Blatt, H.-P., "Nicht-Lineare gleichmässige simultanapproximation," JOAT 8 (1973), 210-248 (Ph.D. dissertation, Universität des Saarlandes, Saarbrucken, 1970).

_______, "Über rationale Tschebyscheff-approximation mehrerer Funktionen," JOAT 9 (1973), 126-148.

Bredendiek, E.,"Simultanapproximationen," Archive for Rational Mechanics and Anal. 33 (1969), 307-330.

Brondsted, A., "A Note on Best Simultaneous Approximation in Normed Linear Spaces," Canad. Math. Bull. 19 (1976), 359-360.

Carroll, M. P., "Simultaneous L_1 Approximation of a Compact Set of Real-valued Functions," Numer. Math. 19 (1972), 110-115.

_______, "On Simultaneous L_1 Approximation," in Approximation Theory (G. G. Lorentz, ed.), Academic Press, New York, 1973, 295-297.

Deutsch, F., Review of the paper by Goel, Holland, Nasim, and Sahney, Math. Reviews, 51#13556.

Diaz, J. B., and H. W. McLaughlin,,"Simultaneous Approximation of a Set of Bounded Real Functions," Math. Comp. 23 (1969), 538-594.

_______, "Simultaneous Chebyshev Approximation of a Set of Bounded Complex-valued Functions," J. Approximation Theory 6 (1969), 419-432.

_______, "On Simultaneous Chebyshev Approximation and Chebyshev Approximation with Chebyshev Approximation with an Additive Weight," J. Approximation Theory 6 (1972), 68-71.

Diliberto, S. P., and E. G. Straus, "On the Approximation of a Function of Several Variables by the Sum of Functions of Fewer Variables," Pacific J. Math. 1 (1951), 195-210.

Dunford, N., and J. T. Schwartz, Linear Operators, Part I, Interscience, New York, 1959.

Dunham, C. B., "Simultaneous Chebyshev Approximation of Functions on an Interval," Proc. Amer. Math. Soc. 18 (1967), 472-477.

_______, "Simultaneous Approximation of a Set," J. Approximation Theory 21 (1977), 205-208.

Dyn, R., "A Straightforward Generalization of Diliberto and Straus' Algorithm Does Not Work," report dated December 1978, Math. Research Center, Madison, Wisconsin.

Edwards, R.E., Functional Analysis, Holt, Rinehart, and Winston, New York, 1965.

Fulkerson, D. R., and P. Wolfe, "An Algorithm for Scaling Matrices," SIAM Review 4 (1962), 142-146.

Gillotte, M. J., and H. W. McLaughlin, "On Additive Weight Approximation and Simultaneous Chebyshev Approximation Using Varisolvent Families," J. Approximation Theory 17 (1976), 35-43.

Goel, D. S., A. S. B. Holland, C. Nasim, B. N. Sahney, "Characterization of an Element of Best LP-simultaneous Approximation," S. R. Ramanujan Memorial Volume, Madras, 1974, 10-14.

_______, "On Best Simultaneous Approximation in Normed Linear Spaces," Canad. Math. Bull. 17 (1974), 523-527.

v. Golitschek, M., "An Algorithm for Scaling Matrices and Computing the Minimum Cycle Mean in a Digraph," preprint, August 1979, Institute for Applied Mathematics, University of Würzburg, Germany.

_______, "Approximation of Functions of Two V riables by the Sum of Two Functions of One Variable," preprint, August 1979, Institute for Applied Mathematics, University of Würzburg, Germany.

Golomb, M., "On the Uniformly Best Approximation of Functions Given by Incomplete Data," Report 121, 1959, Math. Research Center, University of Wisconson, Dec. 1959.

_______, "Approximation by Functions of Fewer Variables," in On Numerical Approximation (R. Langer, ed.), 275-327, University of Wisconsin Press, Madison, Wisconsin, 1959.

_______, "Lectures on Theory of Approximation," Argonne National Laboratory, 1962, 264 pp.

Gordon, W. J., "Blending-function Methods of Bivariate and Multivariate Interpolation and Approximation," Report 834, General Motors Research Labs., October 1968.

Hammerlin, G., and L. L. Schumaker, "Procedures for Kernel Approximation and Solution of Fredholm Integral Equations of the Second Kind," CNA Report 128, Center for Numerical Analysis, UT Austin, November 1977.

Holland, A.S.B., B. N. Sahney, J. Tzimbalario, "On Best Simultaneous Approximation," J. Approximation Theory 17 (1976), 187-188.

Holland, A.S.B., J. H. McCabe, G. M. Phillips, B. N. Sahney, "Best Simultaneous L_1 Approximations," to appear, J. Approximation Theory.

Holland, A.S.B., B. N. Sahney, and J. Tzimbalario, "On Best Simultaneous Approximation," J. Indian Math. Soc. 40 (1976), 69-73.

Holland, A.S.B., and B. N. Sahney, "Some Remarks on Best Simultaneous Approximation," in Theory of Approximation with Applications (A.G.Law, B.N.Sahney, eds.) Academic Press, 1976, 332-337.

James, R. C., "Orthogonality and Linear Functionals in Normed Linear Spaces," Trans. Amer. Math. Soc. 61 (1947), 265-292.

Kelley, C. T., "A Note on the Approximation of Functions of Several Variables by Sums of Functions of One Variable," Report 1873, Math. Research Center, Madison, Wisconsin, August 1978.

Krabs, W., Optimierung und Approximation, B. G. Teubner, Stuttgart, 1975.

Laurent, P. J., and P. D. Tuan, "Global Approximation of a Compact Set of Elements of a Convex Set in a Normed Space," Numer. Math. 15 (1970), 137-150.

Lim, K.-P., "Simultaneous Approximation of Compact Sets of Elements of Convex Sets in Normed Linear Spaces," J. Approximation Theory 12 (1974), 332-351.

Ling, W. H., "On Simultaneous Chebyshev Approximation in the Sum Norm," Proc. Amer. Math. Soc. 48 (1975), 185-188.

Ling, W. H., and H. W. McLaughlin, "Approximation of Random Functions," J. Approximation Theory 20 (1977), 10-22.

Milman, P. D., "On Best Simultaneous Approximation in Normed Linear Spaces," J. Approximation Theory 20 (1977), 223-238.

Phelps, R. R., "Uniqueness of Hahn-Banach Extensions and Unique Best Approximation," Trans. Amer. Math. Soc. 95 (1960), 238-255.

Phillips, G. M., J. H. McCabe, E. W. Cheney, "A Mixed-norm Bivariate Approximation Problem with Applications to Lewanowicz Operators," in Multivariate Approximation (D. C. Handscomb, ed.), Academic Press, New York, 1978. Also CNA Report 127, Center for Numerical Analysis, UT Austin, October 1977.

Phillips, G. M., and B. N. Sahney, "Best Simultaneous Approximation in the L_1 and L_2 Norms," in Theory of Approximation with Applications (A. G. Law and B. N. Sahney, eds.), Academic Press, 1976, 213-218.

Remes, E., "Sur la determination des polynomes d'approximation de degré donné," Comm. Soc. Math. Kharkov, Series 4, 10 (1934), 41-63.

Respess, J., "The Theory of Best Approximation in Tensor-product Spaces," Ph.D. dissertation in progress, University of Texas at Austin.

Robertson, A. P., and Wendy Robertson, "Topological Vector Spaces," Cambridge University Press, Cambridge, 1964.

Rothblum, U. G., and H. Schneider, "Characterizations of Optimal Scalings of Matrices," Report RS 2678 (1978), Yale University, New Haven.

Royden, H. L., Real Analysis, Macmillan, London, 1963.

Rudin, W., Real and Complex Analysis, 2nd. edition, McGraw-Hill Book Company, New York, 1966.

Ruston, A. F., "Auerbach's Theorem ...," Proc. Camb. Phil. Soc. 58 (1964), 476-480.

Settari, A., and K. Aziz, "A Generalization of the Additive Correction Methods for the Iterative Solution of Matrix Equations," SIAM J. on Numer. Anal. 10 (1973), 506-521.

Schaeffer, H. H., Topological Vector Spaces, MacMillan Company, New York, 1966.

Schatten, Robert, "A Theory of Cross-spaces," Annals of Mathematics Studies, No. 26, Princeton University Press, Princeton, 1950.

Sullivan, F., "A Generalization of Best Approximation Operators," Ann. Mat. Pura, Appl. 107 (1975), 245-261.

Usow, K. H., "On L_1-approximation: Computation for Continuous Functions and Continuous Dependence," SIAM J. Numer. Anal. 4 (1967), 70-88.